

Viscous hydrodynamics



Pasi Huovinen

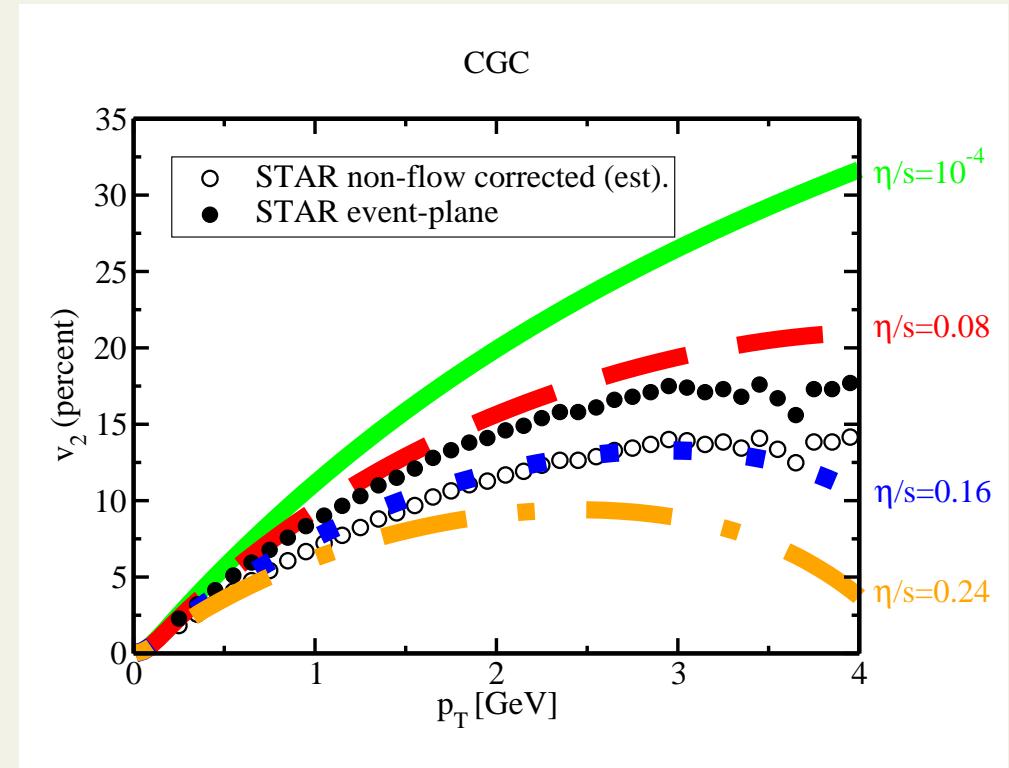
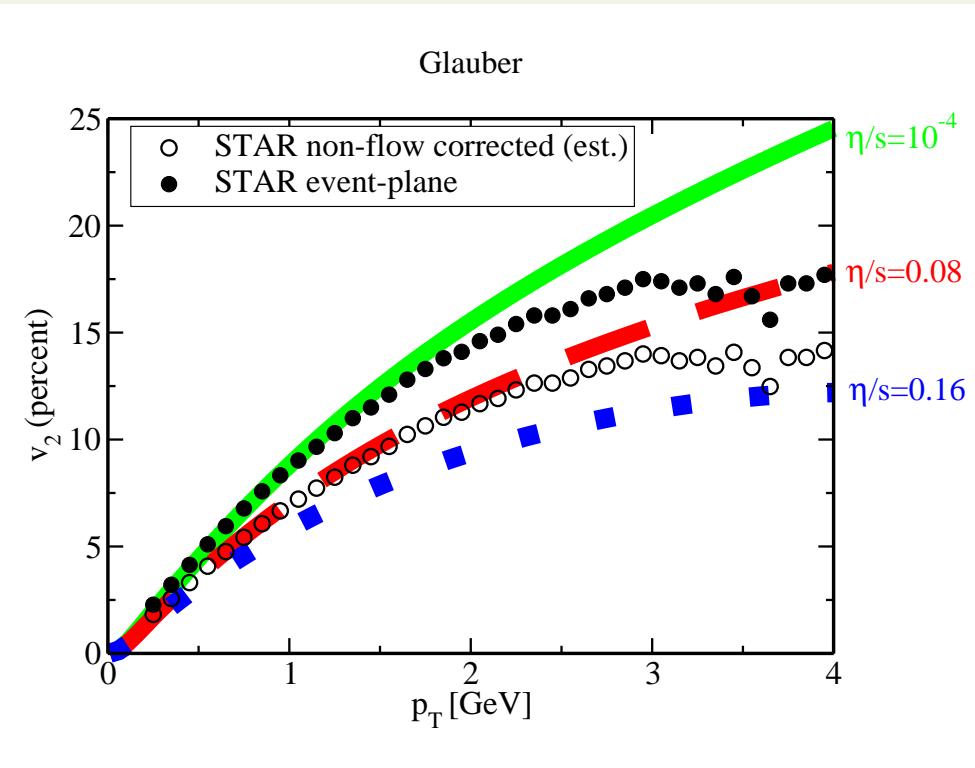
J. W. Goethe Universität

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State of the art

- Luzum & Romatschke, Phys.Rev.C78:034915,2008



- $\eta/s = 0.08$ to 0.16 depending on the initial state

Needs work

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- Equation of state
 - especially hadronic chemistry

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 - $\eta/s = \eta/(T)$

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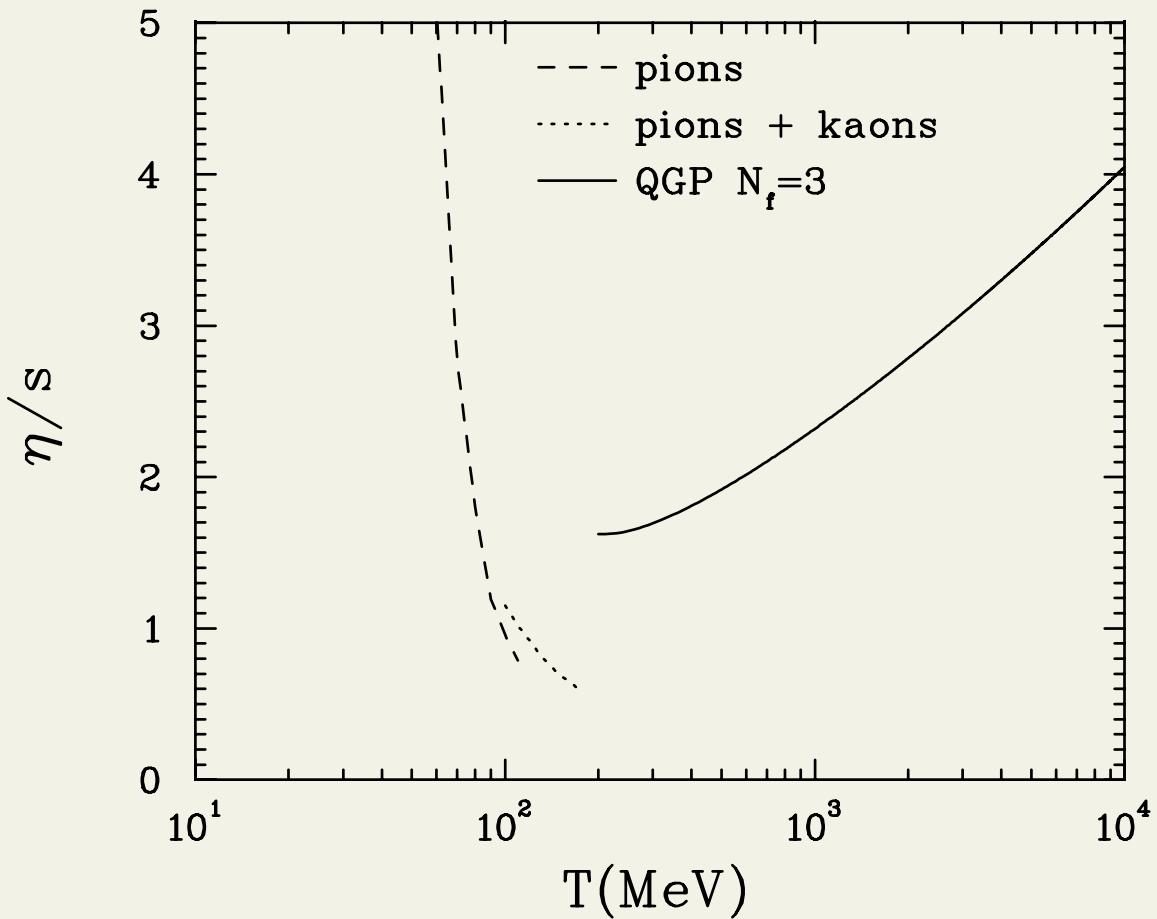
- Initial state
- Equation of state
 - especially hadronic chemistry
- Bulk viscosity
- $\eta/s = \eta/(T)$
- δf , corrections to thermal distributions

Needs work

- Initial state
- Equation of state
 - especially hadronic chemistry
- Bulk viscosity
- $\eta/s = \eta/(T)$
- δf , corrections to thermal distributions
- applicability?

$\eta/s(T)$

Kapusta, McLerran and Csernai, nucl-th/0604032:

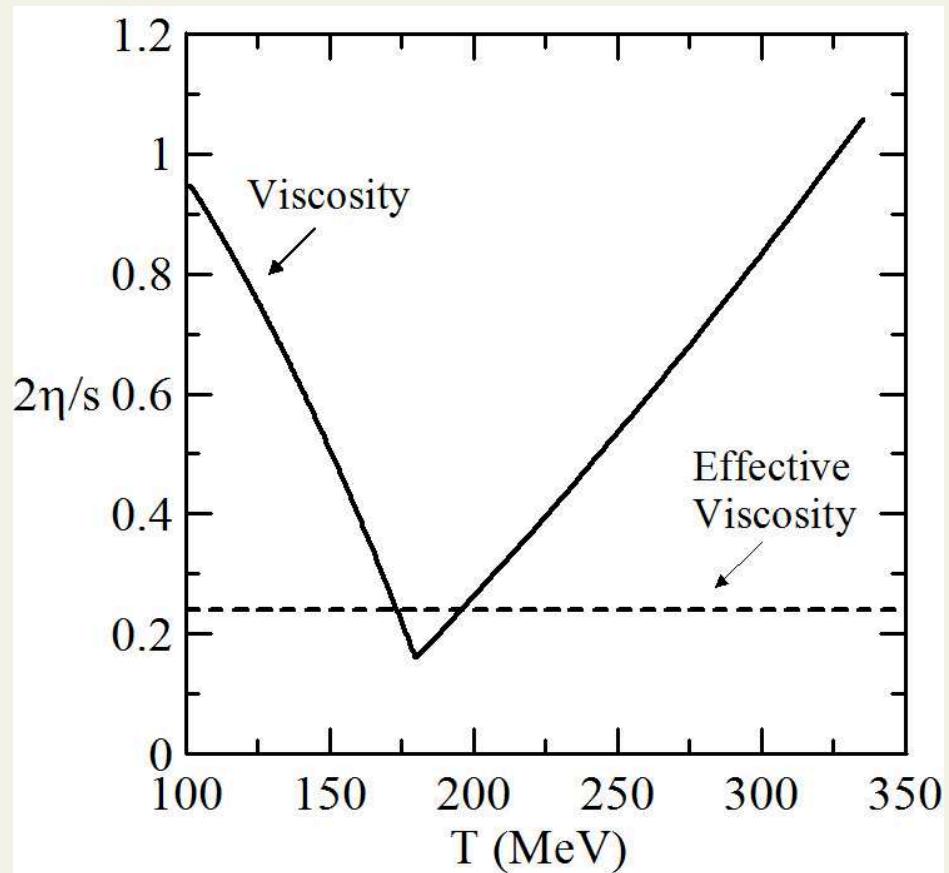


Low T (*Prakash et al.*) using experimental data for 2-body interactions

High T (*Yaffe et al.*) using perturbative QCD

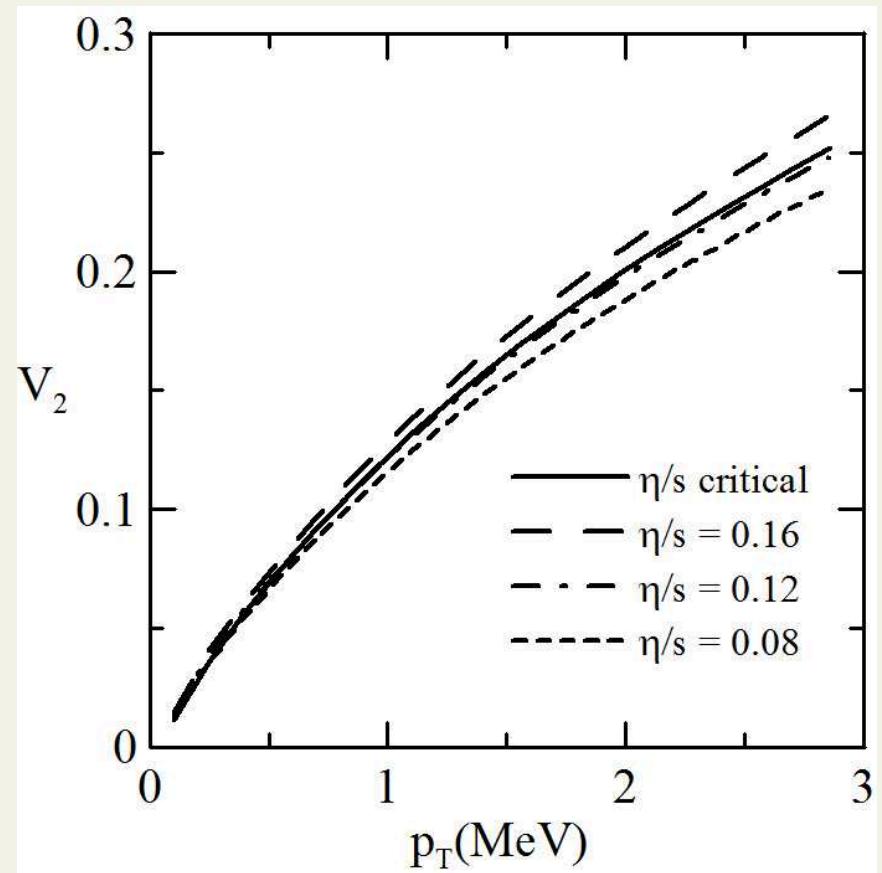
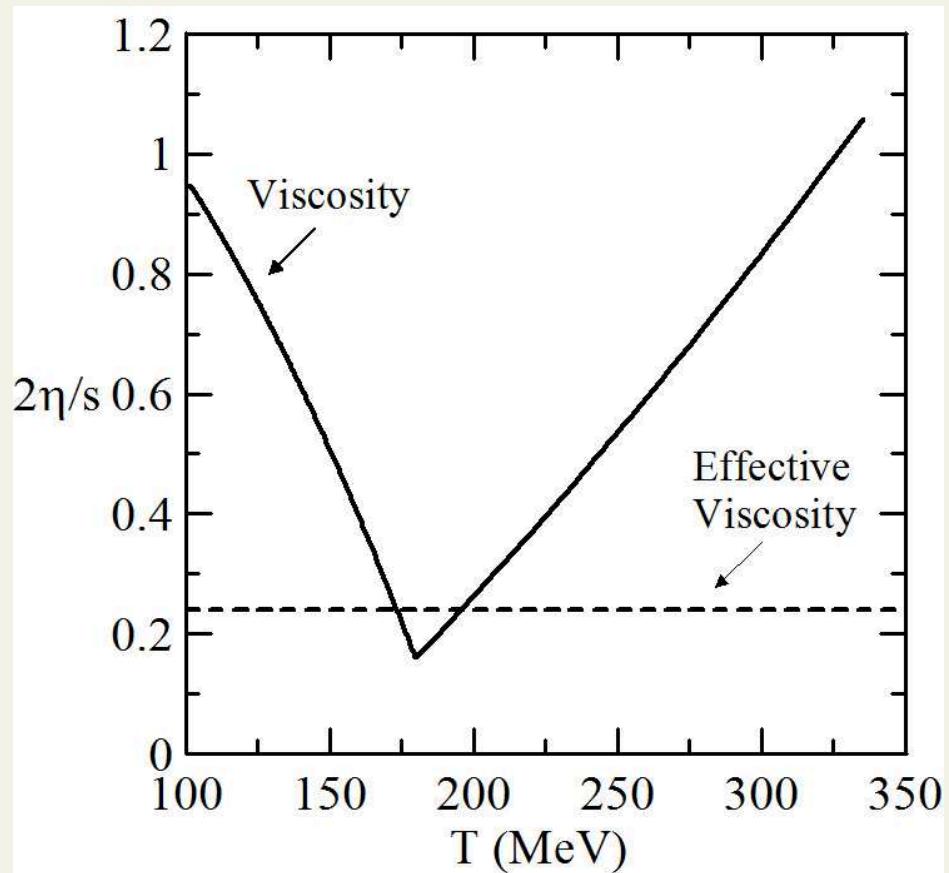
$\eta/s = \eta/s(T)$ vs. $\eta/s = \text{const.}$

Denicol, Kodama and Koide, arXiv:1002.2394:



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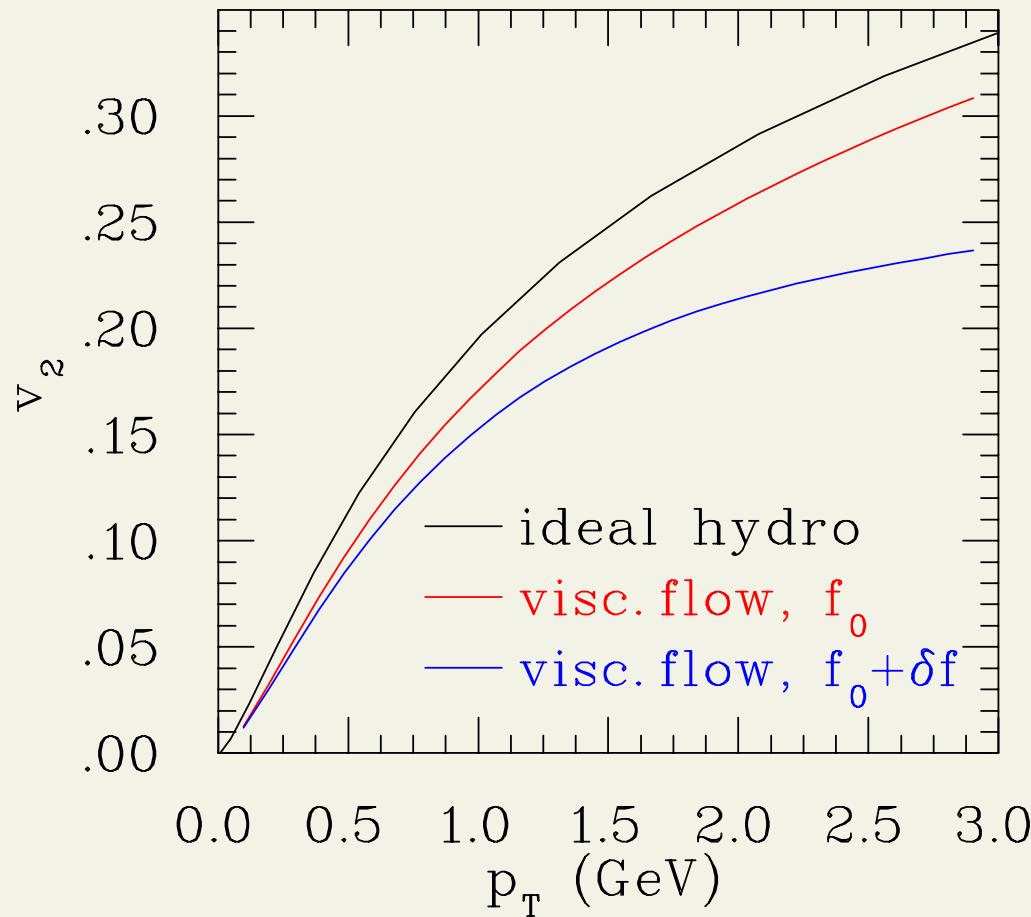
• Indistinguishable!

δf

- TWO effects:**
- dissipative corrections to hydro fields u^μ, T, n
 - dissipative corrections to thermal distributions $f \rightarrow f_0 + \delta f$

$$\eta/s \approx 1/(4\pi) \quad (\sigma_{\text{tr}} \propto \tau^{2/3})$$

$$\delta f = f_0 \left[1 + \frac{p^\mu p^\nu \pi_{\mu\nu}}{8nT^3} \right]$$



Grad 14-moment ansatz

Single particle distribution function

$$f = f_0(1 + \epsilon + \epsilon_\mu p^\mu + \epsilon_{\mu\nu} p^\mu p^\nu) = f_0 + \delta f$$

Single component system, shear only

$$\epsilon^{\mu\nu} = \frac{\pi^{\mu\nu}}{2(\varepsilon + P)T^2}$$

- No reason why this would hold in multicomponent system
- How good is ansatz $\delta f \propto p^2$?

δf for mixtures

Monnai and Hirano, arXiv:1003.3087

- assume conserved charge dependence in vectorial part

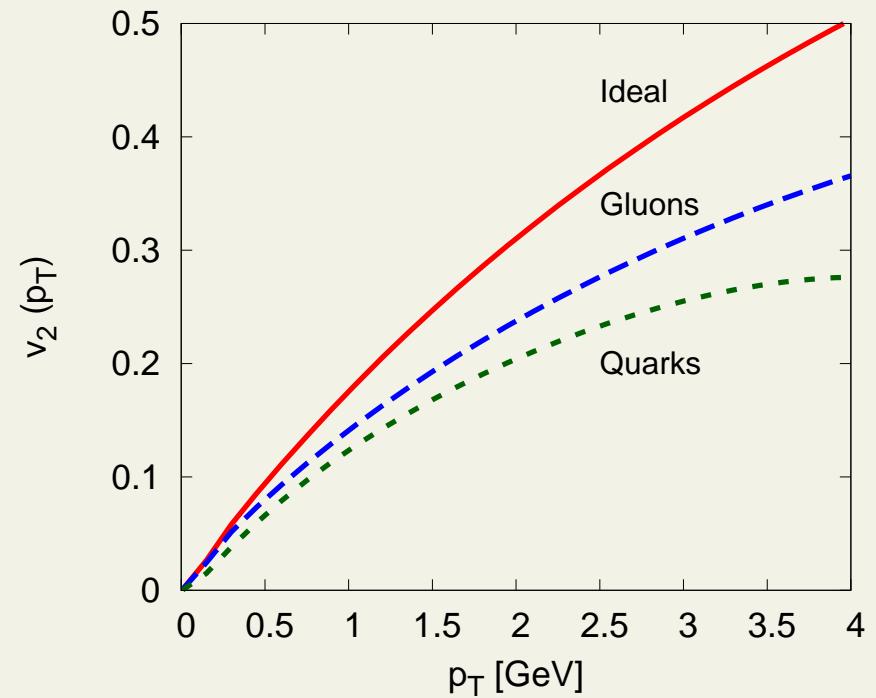
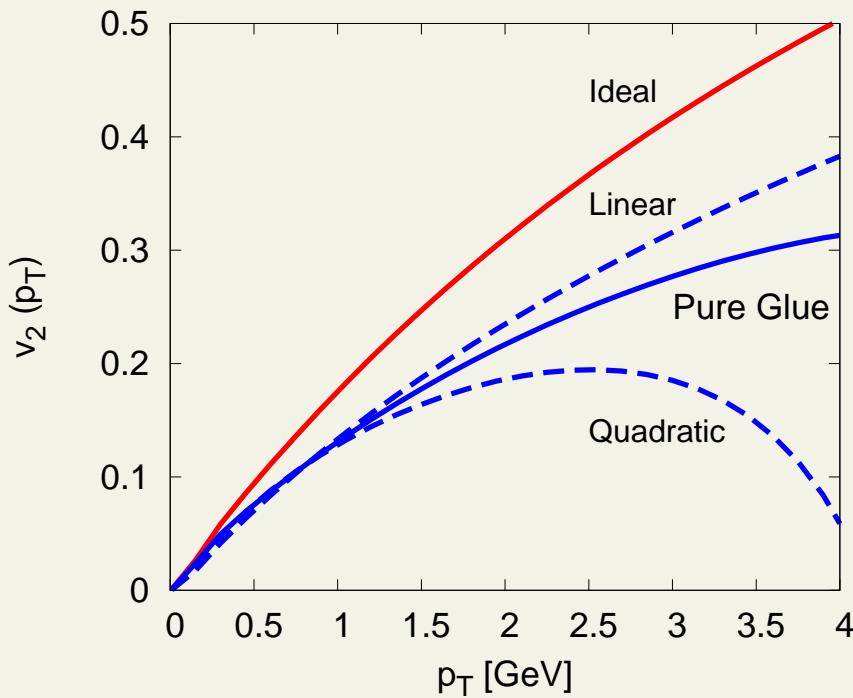
$$\delta f = f_0(p_J^\mu \sum_J q_i^J \epsilon_\mu^J + p_i^\mu p_i^\nu \epsilon_{\mu\nu})$$

- apply Landau matching conditions to fix the unknowns

δf from radiative corrections

Dusling, Moore and Teaney, arXiv:0909.0754

- relaxation time approximation to Boltzmann



- between **linear** and **quadratic**
- should we modify evolution equations for shear too?

IS hydro vs transport

PH & Molnar, PRC 79, 014906:

- start with simplest of all cases - 1D Bjorken (boost invariance)
- $2 \rightarrow 2$ transport \iff conserved particle number
- massless particles \iff ideal gas EoS, $e = 3p$, $\zeta = 0$

$$\pi_{LR}^{\mu\nu} = \text{diag}(0, -\frac{\pi_L}{2}, -\frac{\pi_L}{2}, \pi_L), \quad \Pi \equiv 0, \quad q^\mu \equiv 0 \quad (\text{reflection symm})$$

$$\begin{aligned} \dot{p} + \frac{4p}{3\tau} &= -\frac{\pi_L}{3\tau} \\ \dot{\pi}_L + \frac{\pi_L}{\tau} \left(\frac{2K(\tau)}{3C} + \frac{4}{3} + \frac{\pi_L}{3p} \right) &= -\frac{8p}{9\tau}, \end{aligned}$$

where where

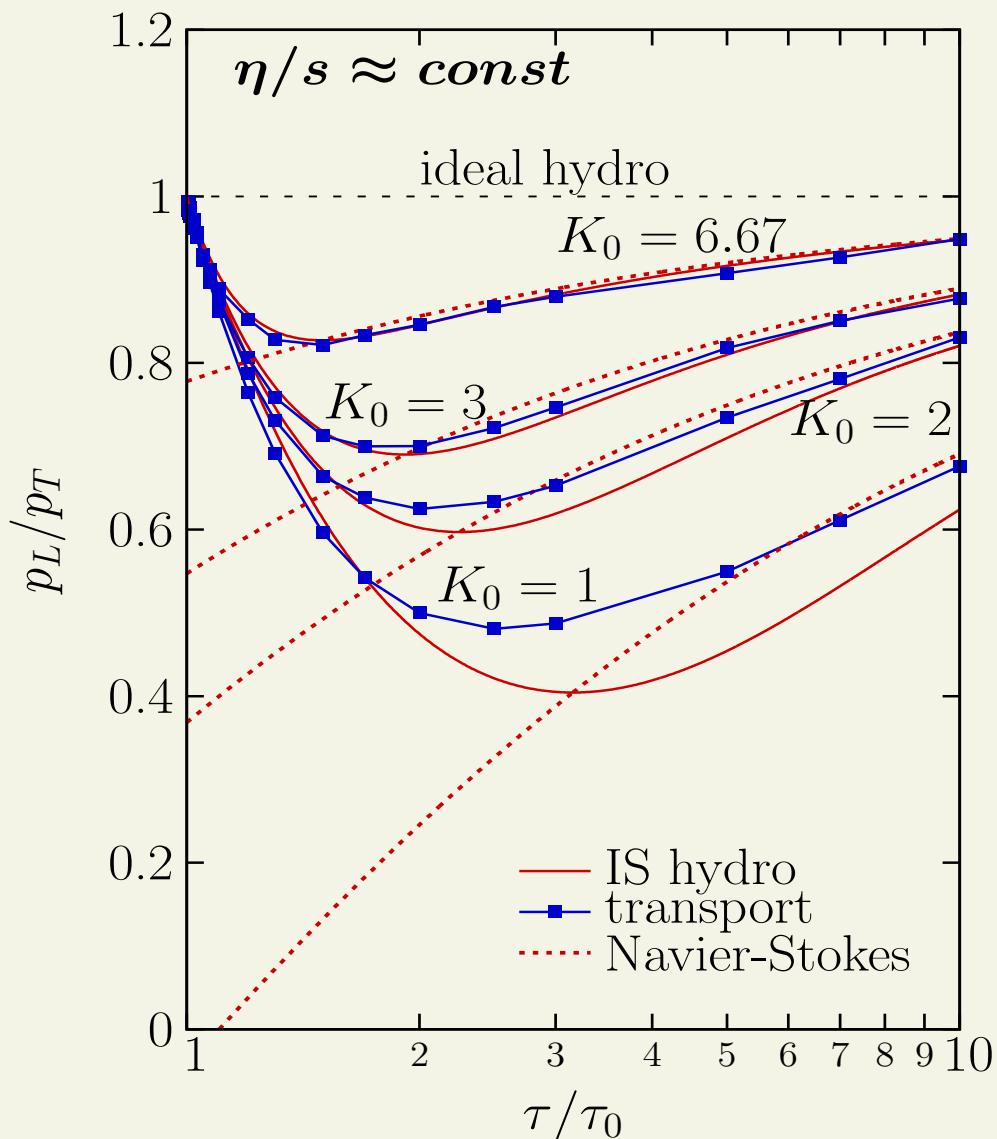
$$K(\tau) \equiv \frac{\tau}{\lambda_{tr}(\tau)} = \frac{\tau_{\text{exp}}}{\tau_{\text{scatt}}} \approx \frac{6}{5} \frac{\tau_{\text{exp}}}{\tau_\pi}$$

is the inverse Knudsen number and $C \approx 0.8$

- i) $\sigma = \text{const: } \lambda_{tr} = 1/n\sigma_{tr} \propto \tau \Rightarrow K = \text{const}$
- ii) $\eta/s \approx \text{const: } \sigma \propto \tau^{2/3} \Rightarrow K = K_0(\tau/\tau_0)^{2/3} \propto \tau^{2/3}$

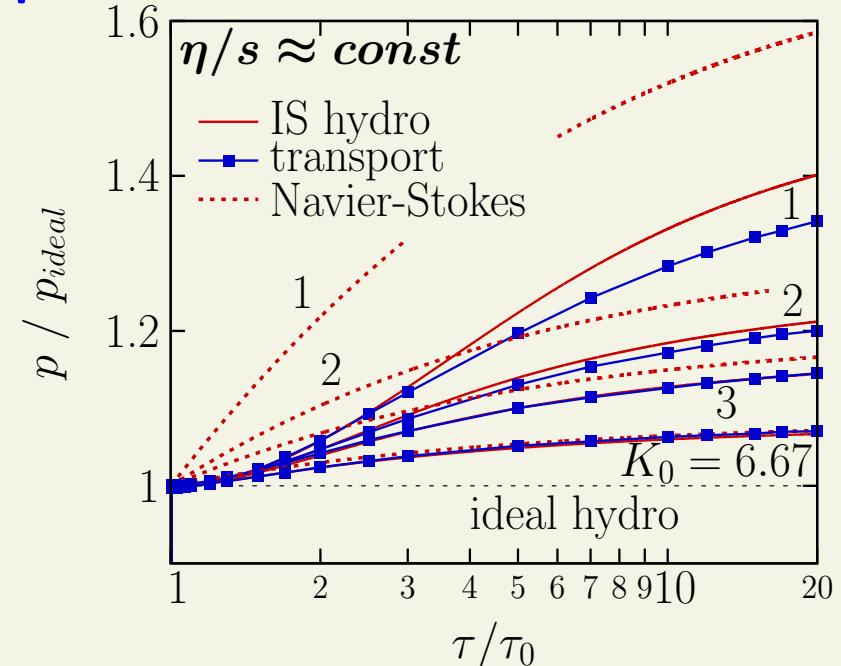
for $\eta/s \approx const$:

pressure anisotropy T_{zz}/T_{xx}

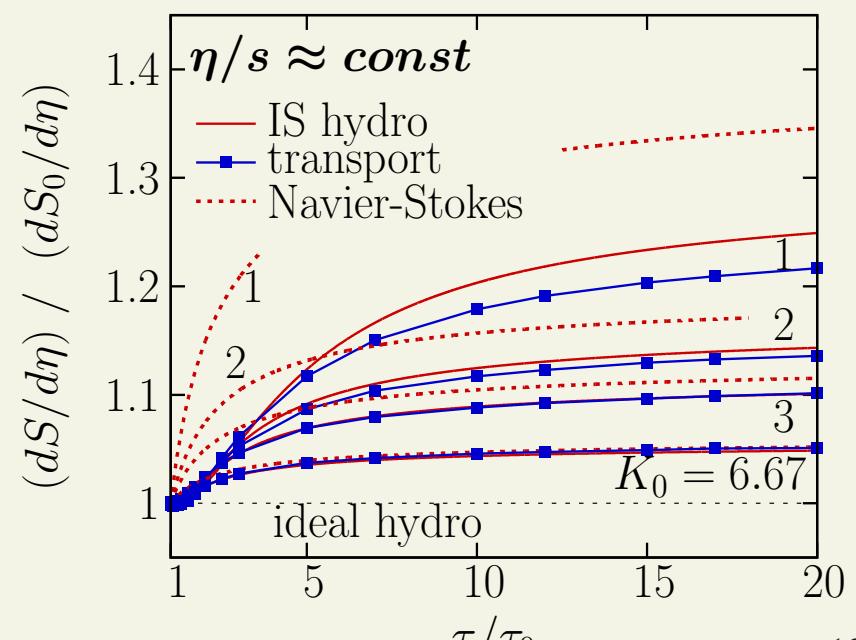


IS hydro 10% accurate when $K_0 \gtrsim 2$

pressure



entropy



Connection to viscosity

$$K = \frac{\tau}{\lambda_{\text{tr}}} = \frac{4Tn\tau}{5\eta} \implies K_0 = \frac{T_0\tau_0 s_0}{5\eta_0}$$

For typical RHIC initial conditions $T_0\tau_0 \sim 1$, therefore

$$K_0 \gtrsim 2 - 3 \implies \frac{\eta}{s} \lesssim \frac{1 - 2}{4\pi}$$

IS hydro applicable only up to a few times the conjectured lower bound!

- The smaller τ_0 , the smaller η/s must be
- When IS hydro is accurate the entropy production is below 20% (necessary condition)

Evolution with bulk

- still simple approach: 1D Bjorken
- lattice + HRG EoS
- $\epsilon_0 = 15 \text{ GeV/fm}^3$ ($T_0 = 297 \text{ MeV}$) at $\tau_0 = 0.6 \text{ fm}$,
scale to other τ_0 via entropy conservation

- ζ/s parametrized as

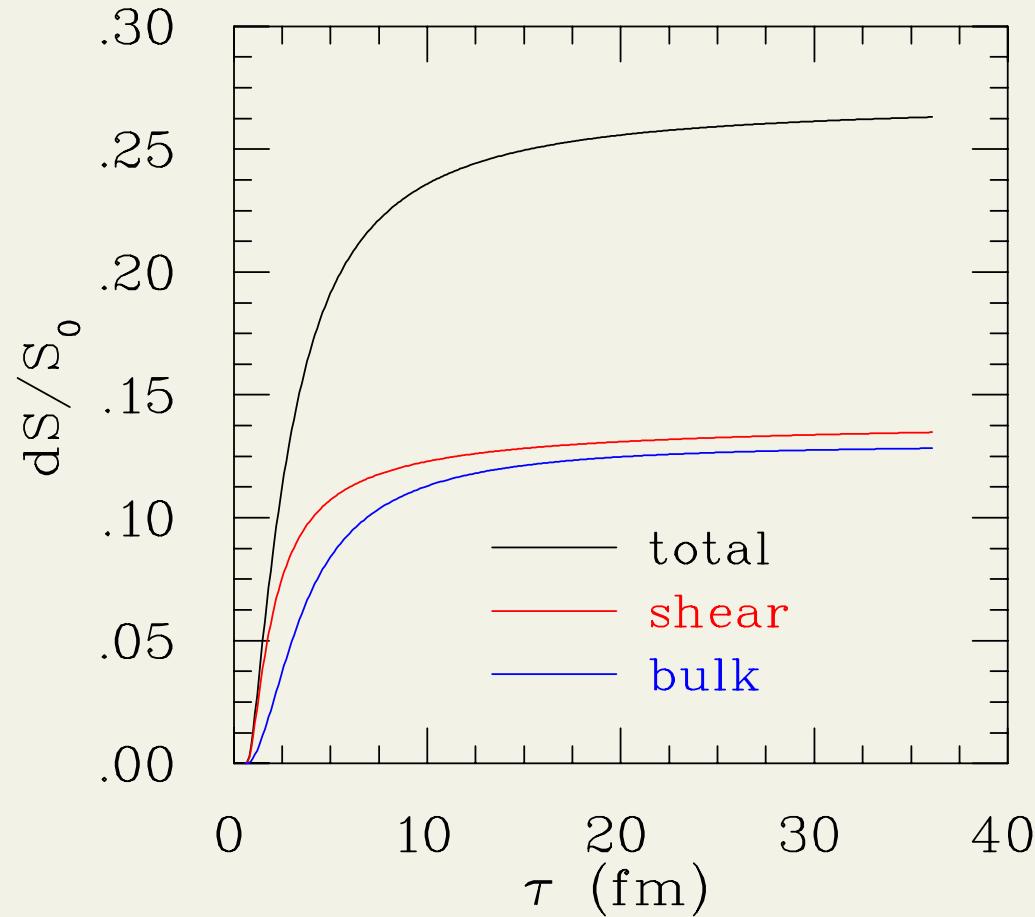
$$\frac{\zeta}{s} = \frac{\zeta_0}{((T - T_\zeta)/\sigma)^2 + 1}$$

with $\zeta_0 = 0.2$, $\sigma = 30 \text{ MeV}$, $T_\zeta = 192 \text{ MeV}$

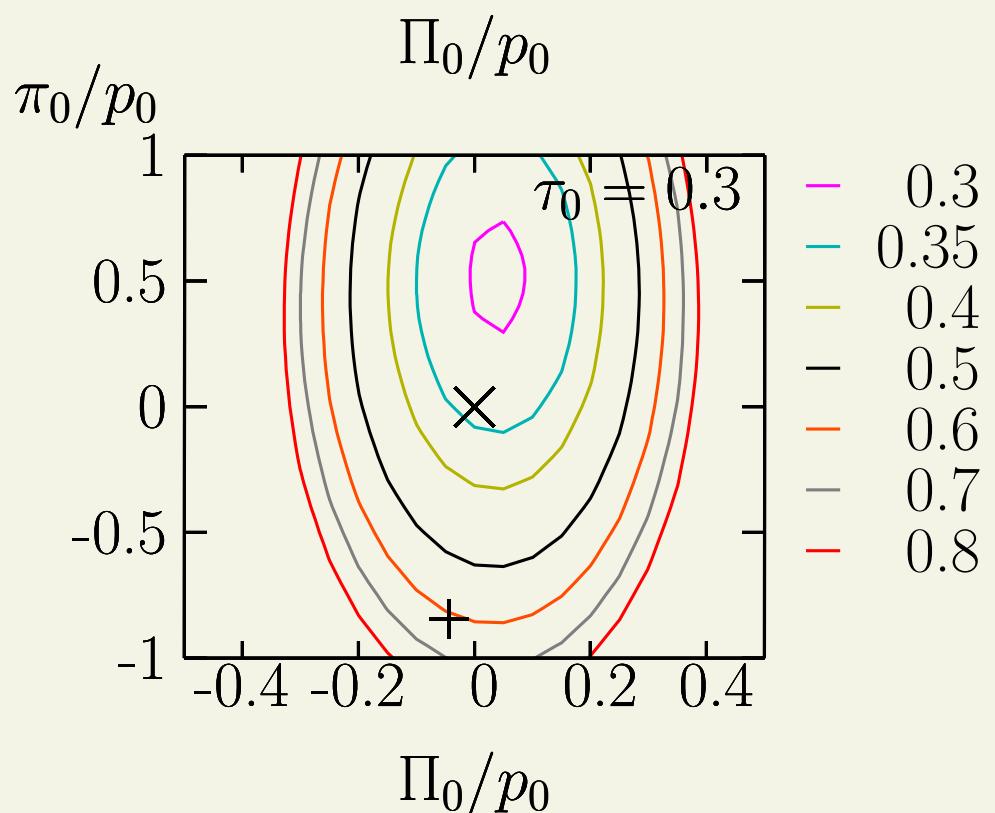
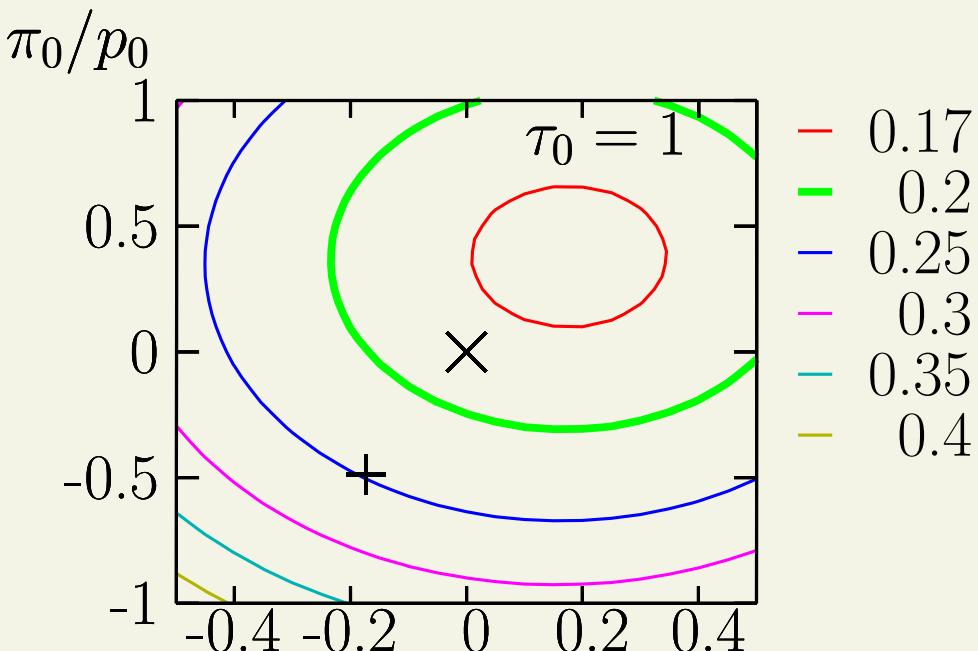
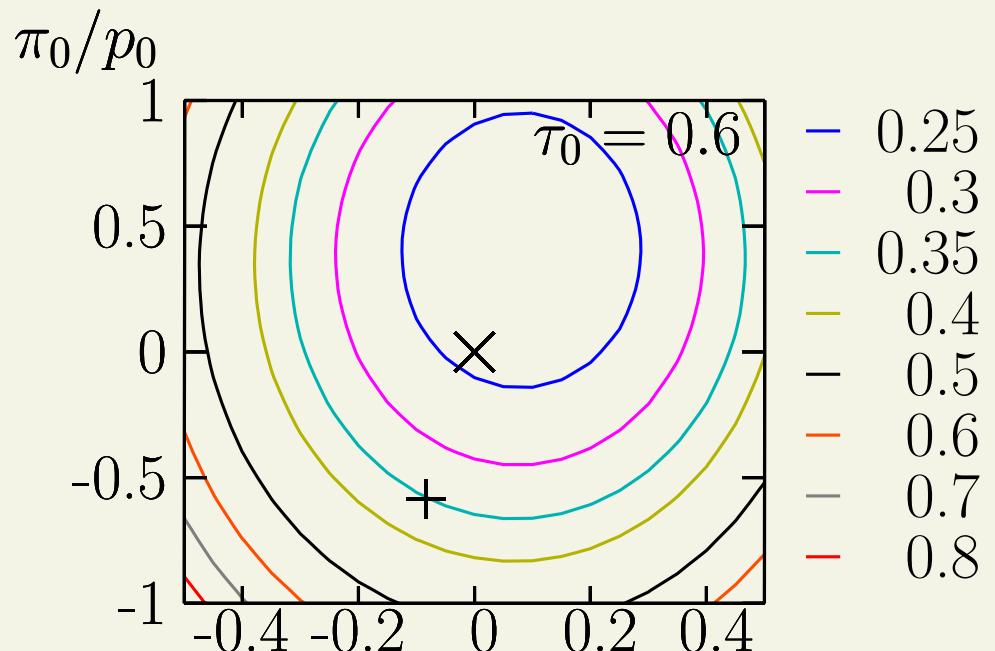
- Minimum shear: $\eta/s = 1/(4\pi)$

- $\tau_\pi = \frac{6s}{T\eta}$, assume $\tau_\pi = \tau_\Pi$

Entropy production



- bulk and shear produce equal amount of entropy!
- depending on the maximum value of ζ/s . . .
- bulk viscosity cannot be ignored
- 20% limit \Rightarrow calculation not valid?



requirement of modest ≤ 20 entropy production severely limits initial conditions if $\tau_0 \lesssim 1$ fm

even Navier-Stokes limit is hard to accommodate

color glass $\pi_0 \sim -p_0$ is out of hydro validity even at $\tau_0 = 1$ fm

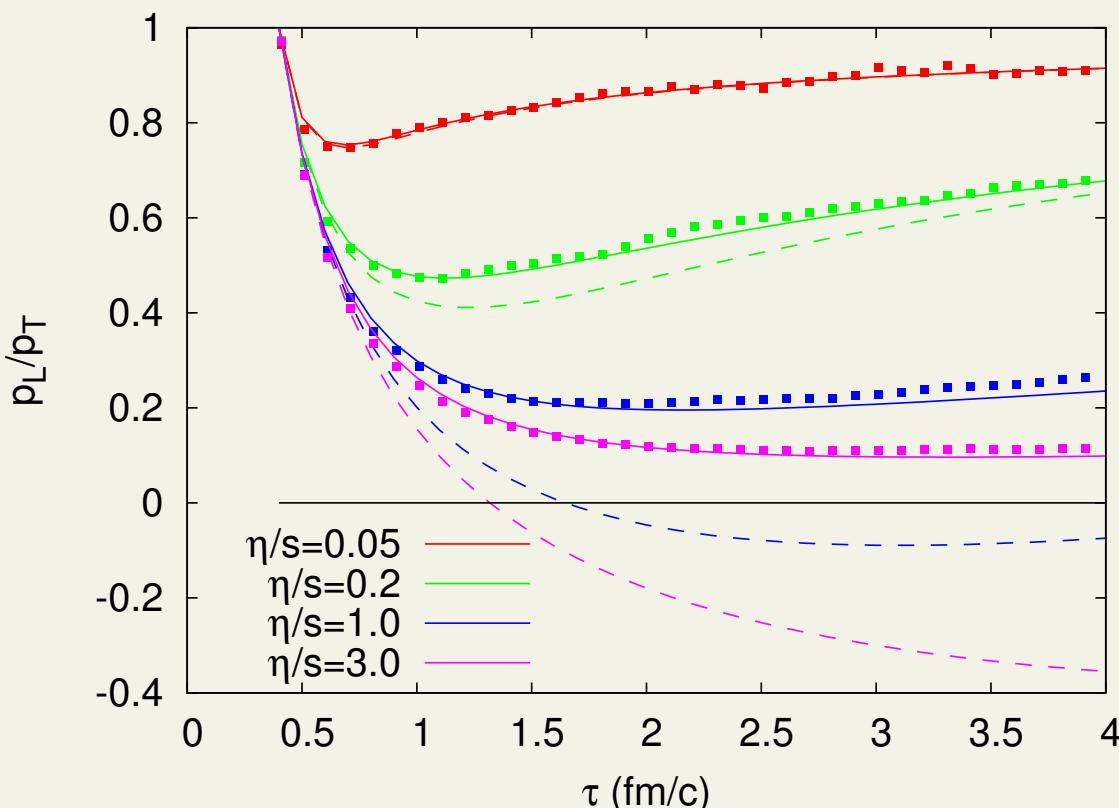
Third order terms

Ei, Xu & Greiner: Phys. Rev. C 81, 041901 (2010)

- include term(s) third order in dissipative quantities:

$$S^\mu = su^\mu - \beta_2 \pi_{\lambda\nu} \pi^{\lambda\nu} \frac{u^\mu}{2T} + \alpha \beta_2^2 \pi_{\alpha\beta} \pi^\alpha{}_\lambda \pi^{\beta\lambda} \frac{u^\mu}{T}$$

Pressure anisotropy in 1D Bjorken:



$$\begin{aligned} \frac{\eta}{s} = 0.05 &\approx \frac{0.6}{4\pi} \Leftrightarrow K \approx 4 \\ \frac{\eta}{s} = 0.2 &\approx \frac{2.5}{4\pi} \Leftrightarrow K \approx 1 \\ \frac{\eta}{s} = 1.0 &\approx \frac{13}{4\pi} \Leftrightarrow K \approx 0.2 \\ \frac{\eta}{s} = 3.0 &\approx \frac{25}{4\pi} \Leftrightarrow K \approx 0.07 \end{aligned}$$

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- very phenomenological
 - coefficients?
- stability questionable

Rethinking kinetic theory

Usual approach:

- Derive Israel-Stewart eqs. from 2nd moment of Boltzmann equation

$$\int \frac{d^3 p}{E} p^\mu p^\nu p^\alpha \partial_\alpha f = \int \frac{d^3 p}{E} p^\mu p^\nu \mathcal{C}[f]$$

Denicol, Koide and Rischke, arXiv:1004.5013:

- Use the definition of shear stress instead:

$$\langle \dot{\pi}^{\mu\nu} \rangle = \int \frac{d^3 p}{E} p^{\langle \mu} p^{\nu \rangle} \delta \dot{f}$$

and Boltzmann equation in the form

$$\delta \dot{f} = -\dot{f}_0 - \frac{1}{u \cdot K} K \cdot \nabla f + \frac{1}{u \cdot K} C[f]$$

Evolution equations

- from 3rd moment (Betz, Henkel and Rischke, J.Phys.G36:064029,2009)

$$\begin{aligned}\pi^{\mu\nu} &= \pi_{\text{NS}}^{\mu\nu} - \tau_\pi \dot{\pi}^{<\mu\nu>} \\ &+ 2\tau_{\pi q} q^{<\mu} \dot{u}^{\nu>} + 2\ell_{\pi q} \nabla^{<\mu} q^{\nu>} + 2\tau_\pi \pi_\lambda^{<\mu} \omega^{\nu>\lambda} - 2\eta \hat{\delta}_2 \pi^{\mu\nu} \theta \\ &- 2\tau_\pi \pi_\lambda^{<\mu} \sigma^{\nu>\lambda} - 2\lambda_{\pi q} q^{<\mu} \nabla^{\nu>} \alpha + 2\lambda_{\pi\Pi} \Pi \sigma^{\mu\nu}\end{aligned}$$

- from projection (Denicol, Koide and Rischke, arXiv:1004.5013)

$$\begin{aligned}\dot{\pi}^{<\mu\nu>} &= -\frac{\pi^{\mu\nu}}{\tau_\pi} + 2\beta_\pi \sigma^{\mu\nu} + 2\pi_\alpha^{<\mu} \omega^{\nu>\alpha} - \tau_{\pi n} n^{<\mu} \dot{u}^{\nu>} \\ &+ \ell_{\pi n} \nabla^{<\mu} n^{\nu>} - \delta_{\pi\pi} \pi^{\mu\nu} \theta - \tau_{\pi\pi} \pi_\alpha^{<\mu} \sigma^{\nu>\alpha} \\ &+ \lambda_{\pi n} n^{<\mu} \nabla^{\nu>} \alpha_0 + \lambda_{\pi\Pi} \Pi \sigma^{\mu\nu}\end{aligned}$$

1D Bjorken

Israel-Stewart can be cast in a form

$$\frac{d\pi}{d\tau} + \frac{\pi}{\tau_\pi} = \beta_\pi \frac{4}{3\tau} - \lambda \frac{\pi}{\tau}$$

where

$$\beta_\pi = \frac{2P}{3}, \quad \tau_\pi^{-1} = \frac{5}{9} \sigma n, \quad \lambda = 2$$

DKR, same equation

$$\frac{d\pi}{d\tau} + \frac{\pi}{\tau_\pi} = \beta_\pi \frac{4}{3\tau} - \lambda \frac{\pi}{\tau}$$

but, where

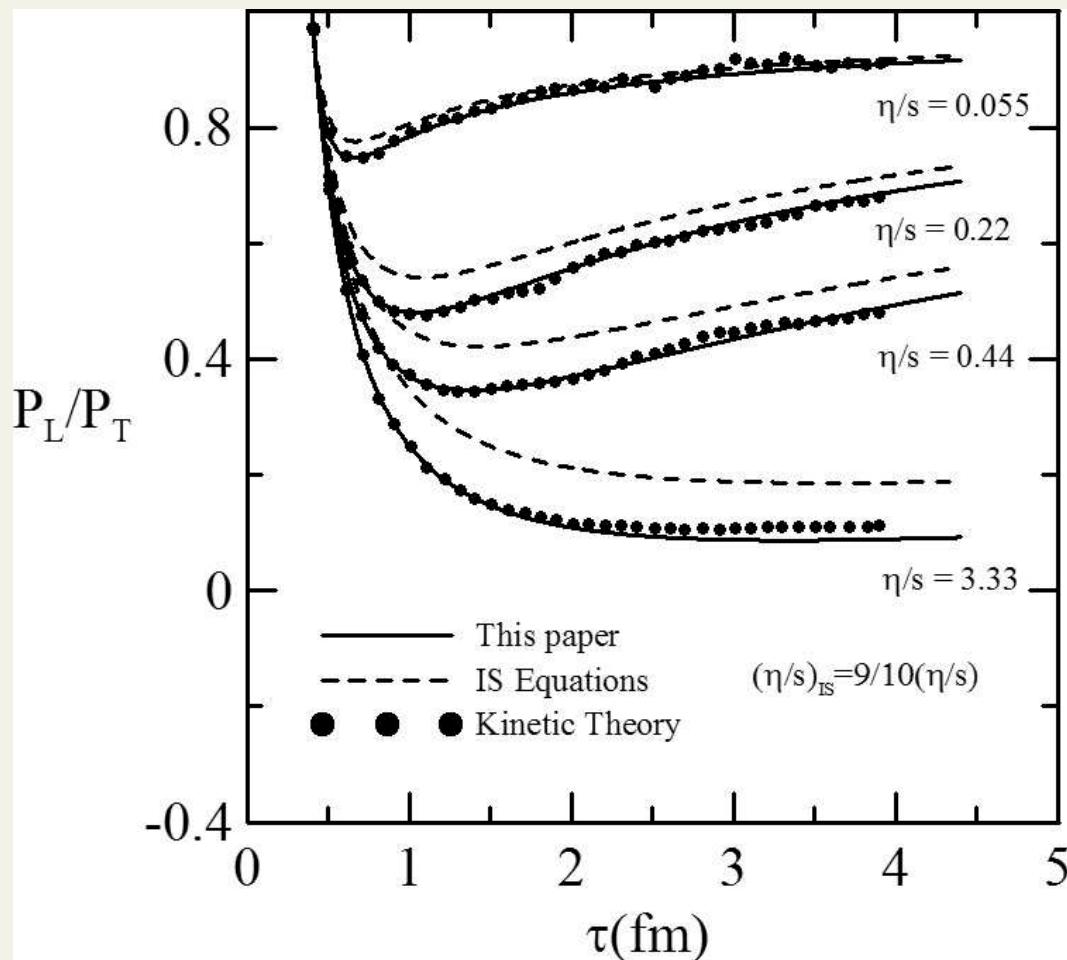
$$\beta_\pi = \frac{4P}{5}, \quad \tau_\pi^{-1} = \frac{3}{5} \sigma n, \quad \lambda = \frac{124}{63}$$

AND

$$\eta_{DKR} = \frac{9}{10} \eta_{NS}$$

Pressure anisotropy

- 1D Bjorken



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- significant improvement!
- only $2 \leftrightarrow 2$ processes so far

Conclusions

- differentiating between $\eta/s = \eta/s(T)$ and $\eta/s = \text{const}$ **difficult**
- **dissipative corrections** to thermal distributions?
- the **region of applicability** for viscous hydro **narrow**
 - new derivation from kinetic theory \Rightarrow **significant improvement**